

# ***Height Reference Systems***

***by***

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***Norwegian Mapping Authority***



# *Main items*

- Geoid*
- Traditional height determination (leveling)*
- Satellite altimetry*
- Tide gauges*
  - Harmonic analysis*
  - Lowest Astronomical Tide, LAT*
- Practical implementation*
- Summary*



# *Definition of the geoid*

*An equipotential surface of the earth's gravity potential  $W$ ,  $W=\text{const}$ .*

*$W=U+T$  where  $U$  is the known normal gravity potential and  $T$  is the disturbing or anomalous potential*

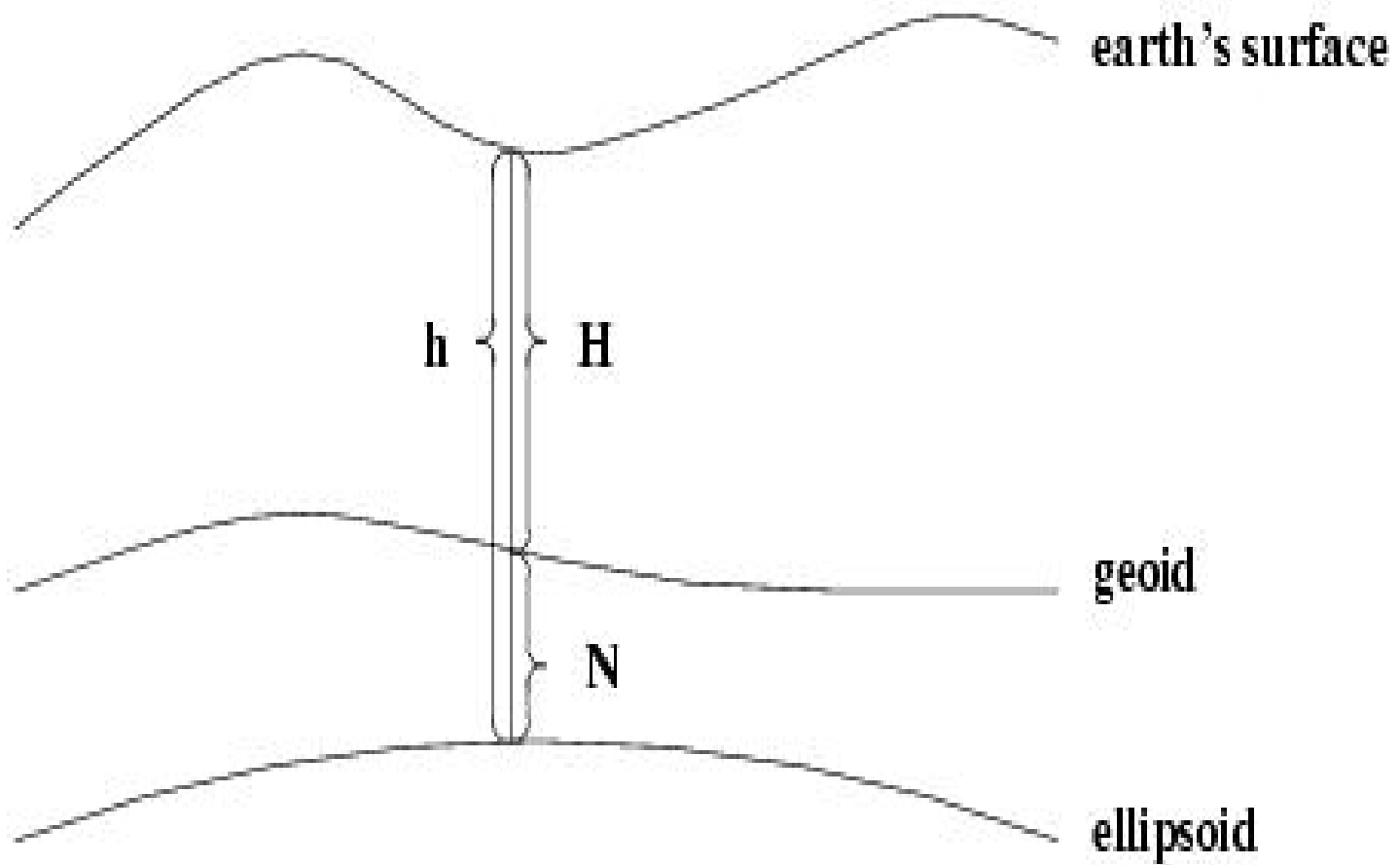
*One of physical geodesy's main tasks is the determination of  $T$*



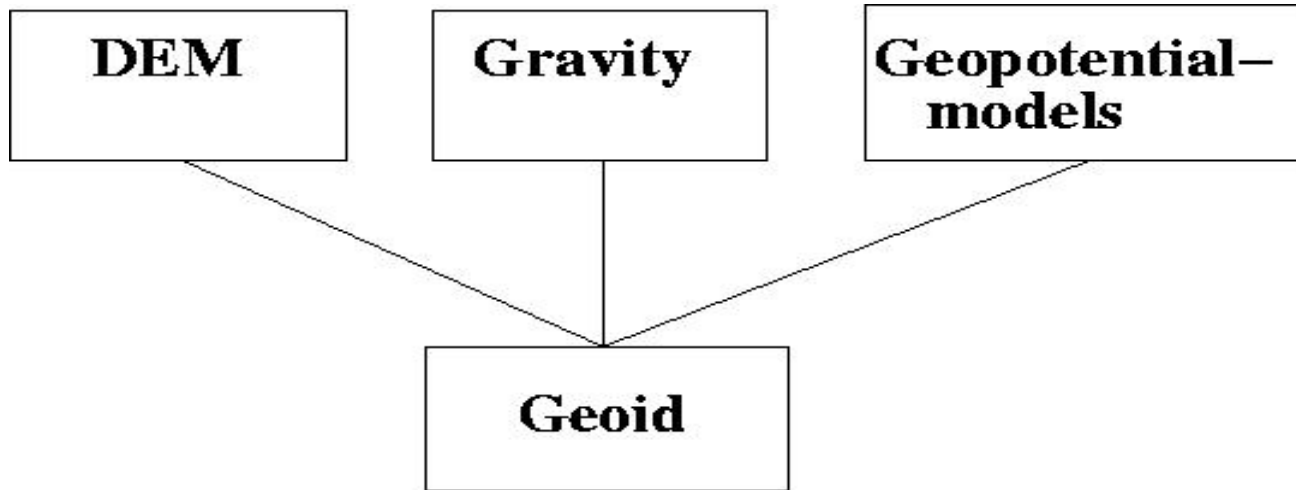
# *The importance of the geoid*

- Classical reference surface*
- Physical meaningful*
- Water flows in the right direction*
- Satellite techniques*
  - Height determination with GPS*
  - Optimal use of satellite altimetry in oceanography*





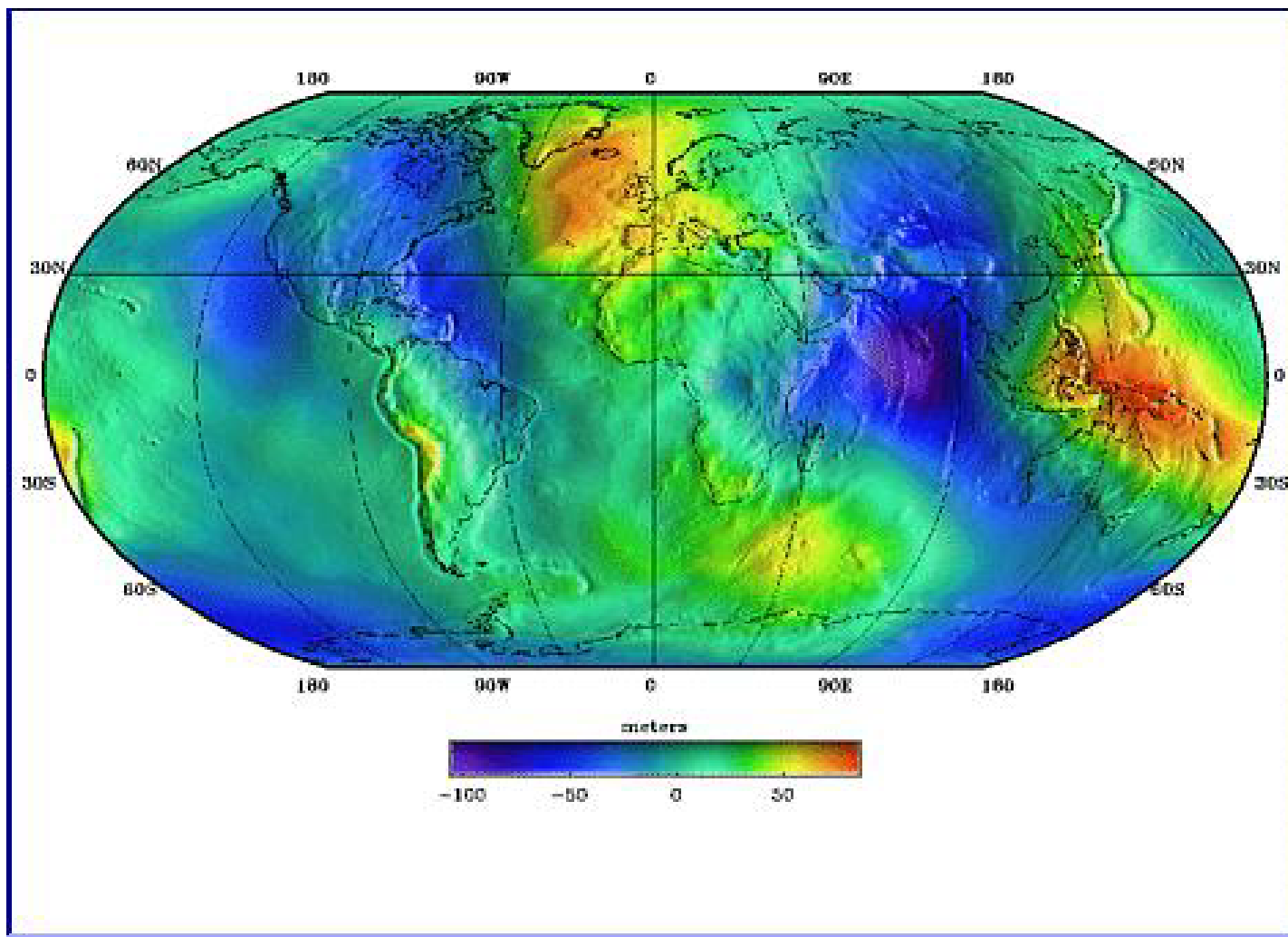
# *Determination of the geoid*



***The geoid is determined by using a remove/restore technique and by solving either integral (FFT) or differential equations (collocation)***



# *EGM96 Geoid*



# *Future enhancements*

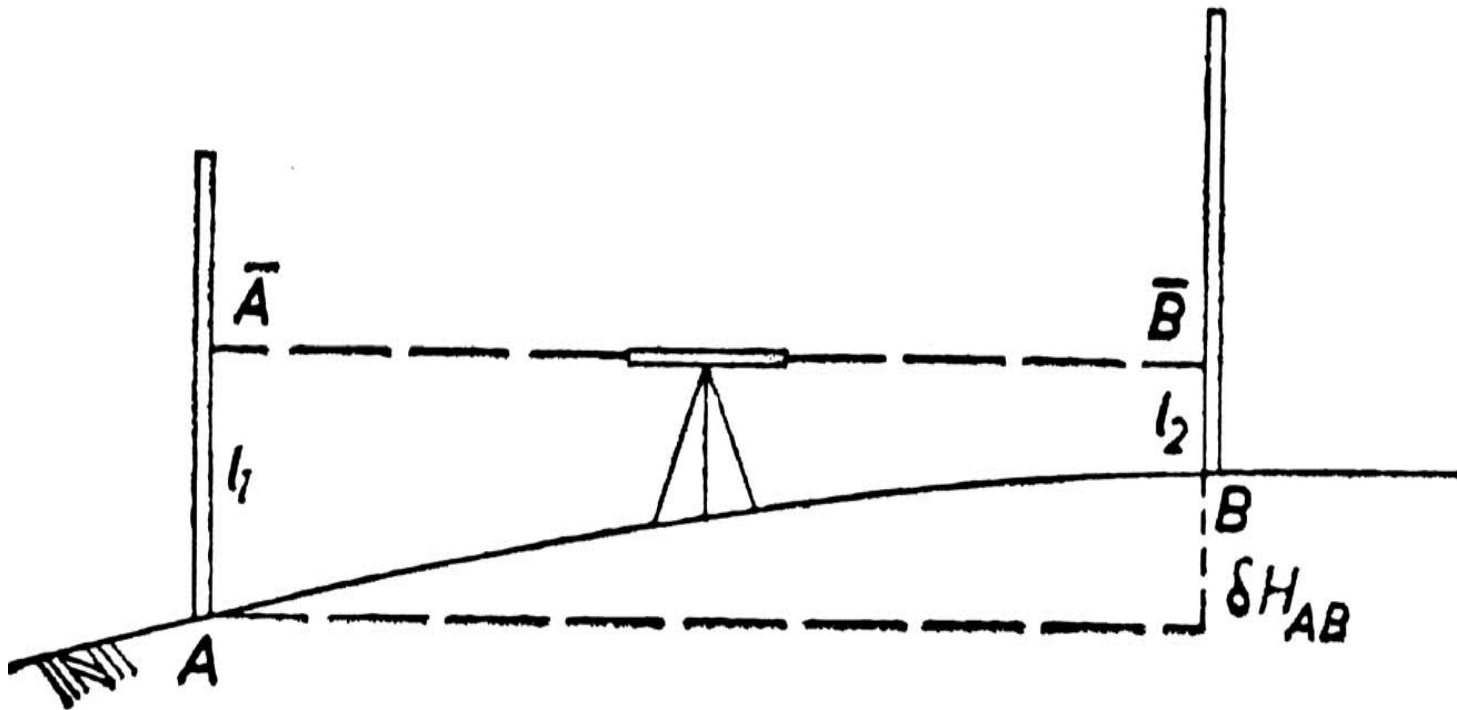
- Improved data coverage and quality*
- Improved geopotential models from the new satellite missions*
  - CHAMP*
  - GRACE*
  - GOCE*
- Theoretical improvements and refinements*
- Digital density models*
- Bathymetry*



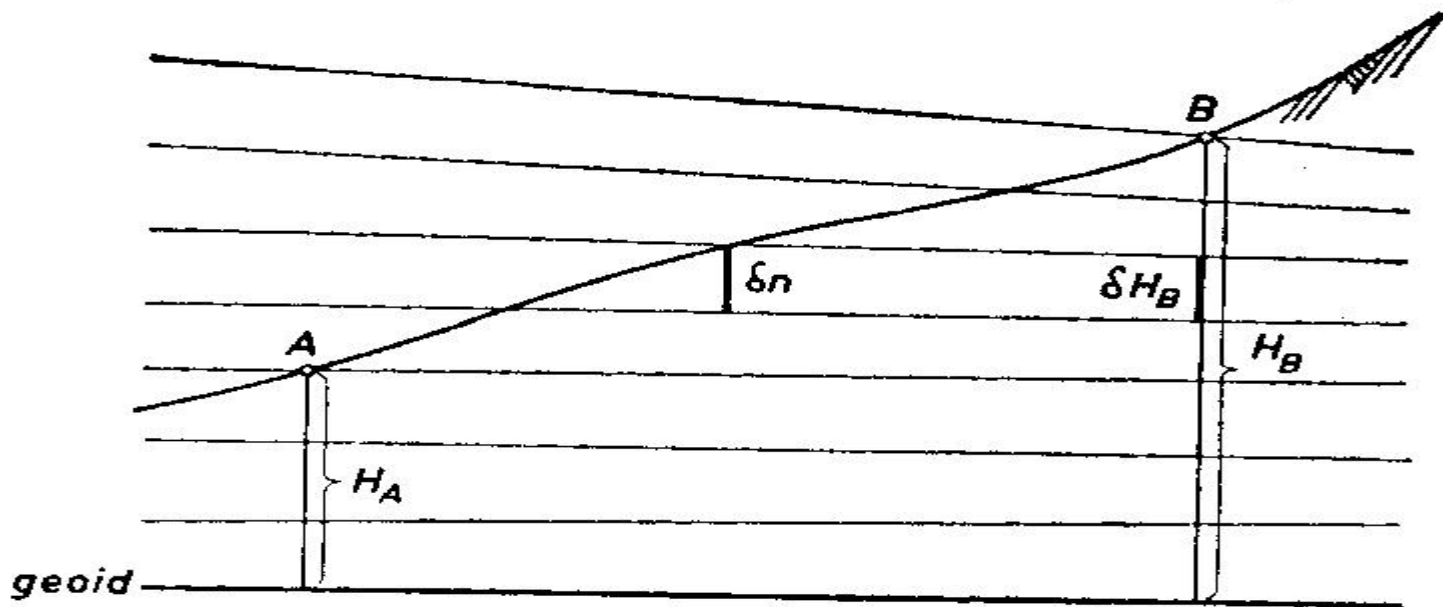


# Leveling

*The height difference between two neighboring points are measured*



***The height difference between the points A and B consists of a sum of leveling measurements***



***As can be seen from the last figure the sum of dn's is not equal to the sum of dH's!***

***Leveling must be combined with gravity measurements,  $g$ , to produce potential differences. If  $O$  is a point on the geoid then the geopotential at  $A$  is defined by***

$$C = \int_O^A g \, dn$$

***The geopotential  $C$  is independent of the particular leveling line used***



# *Different height systems*

***Given geopotential C then the heights can be defined as follows:***

$$\text{Height} = C/G$$

***where we have***

***Orthometric height: G= mean of the gravity g along the plumb line from the point down to the geoid***

***Normal height: G=mean of the normal gravity along the plumb line***

***Dynamic height: G=mean normal gravity for some standard latitude (like 45)***



# ***Some inconveniences***

***Mean gravity is not known***

***Each leveling observation is very accurate, but be aware of small systematic errors that may sum up***

***Leveling instruments do sometimes not function as intended***

***Time dependency, land uplift***

***Very time consuming technique and thus expensive***



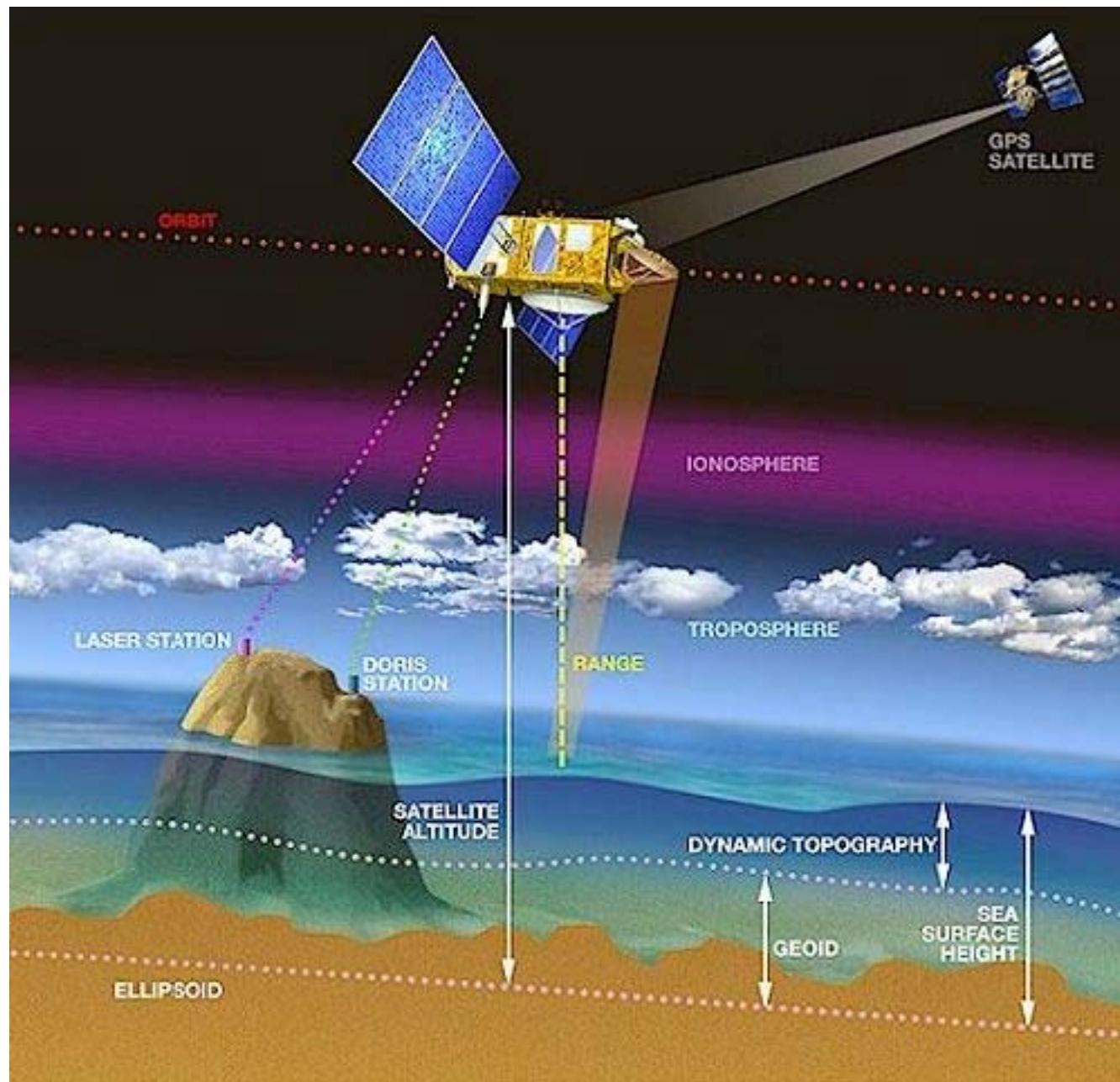
# ***Satellite Altimetry***

***Radar signals emitted from satellites in low earth orbits are being reflected from the earth's surface. Combined with accurate tracking of the satellites this determines the instantaneous surface of the earth. So far the technique has primarily been used to map the ocean surface.***

***See also:***

***[http://www.avisioceanobs.com/html/alti/welcome\\_uk.html](http://www.avisioceanobs.com/html/alti/welcome_uk.html)***





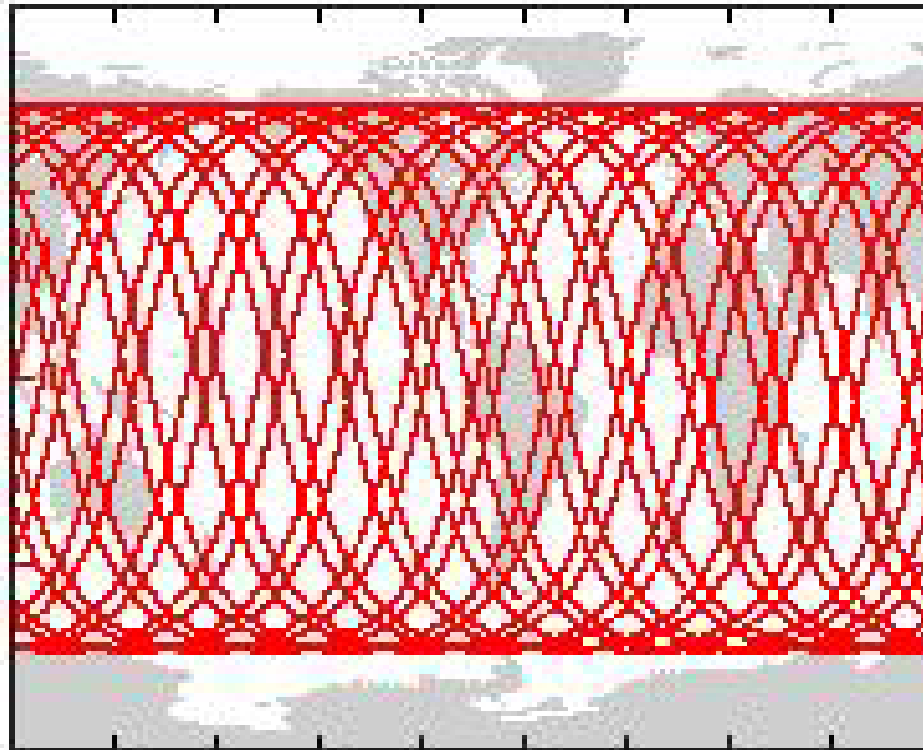
# *Altimetry satellites*

<i>Satellite</i>	<i>Launch date</i>	<i>Orbital height (km)</i>
<i>Seasat</i>	<i>June 26, 1978</i>	<i>800</i>
<i>Geosat</i>	<i>March 12, 1985</i>	<i>800</i>
<i>ERS-1</i>	<i>July 17, 1991</i>	<i>800</i>
<i>Topex/Poseidon</i>	<i>August 10, 1992</i>	<i>1300</i>
<i>ERS-2</i>	<i>April 21, 1995</i>	<i>800</i>
<i>GFO</i>	<i>February 10, 1998</i>	<i>880</i>
<i>Jason-1</i>	<i>December 7, 2001</i>	<i>1300</i>
<i>Envisat</i>	<i>March 1, 2002</i>	<i>800</i>



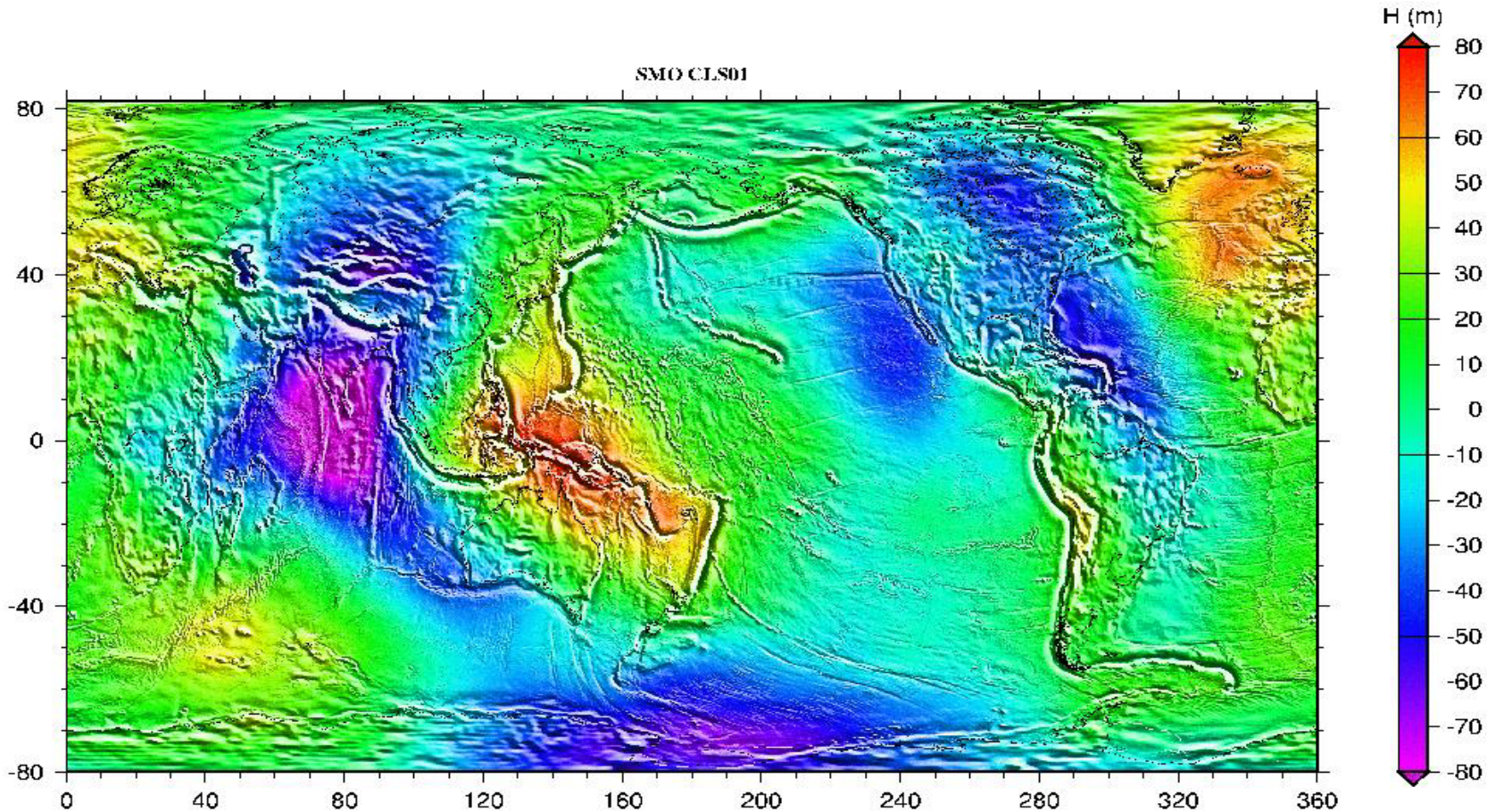


# *Altimetry tracks*



**TOPEX/POSEIDON altimeter tracks**

# CLS 01 Mean Sea Surface

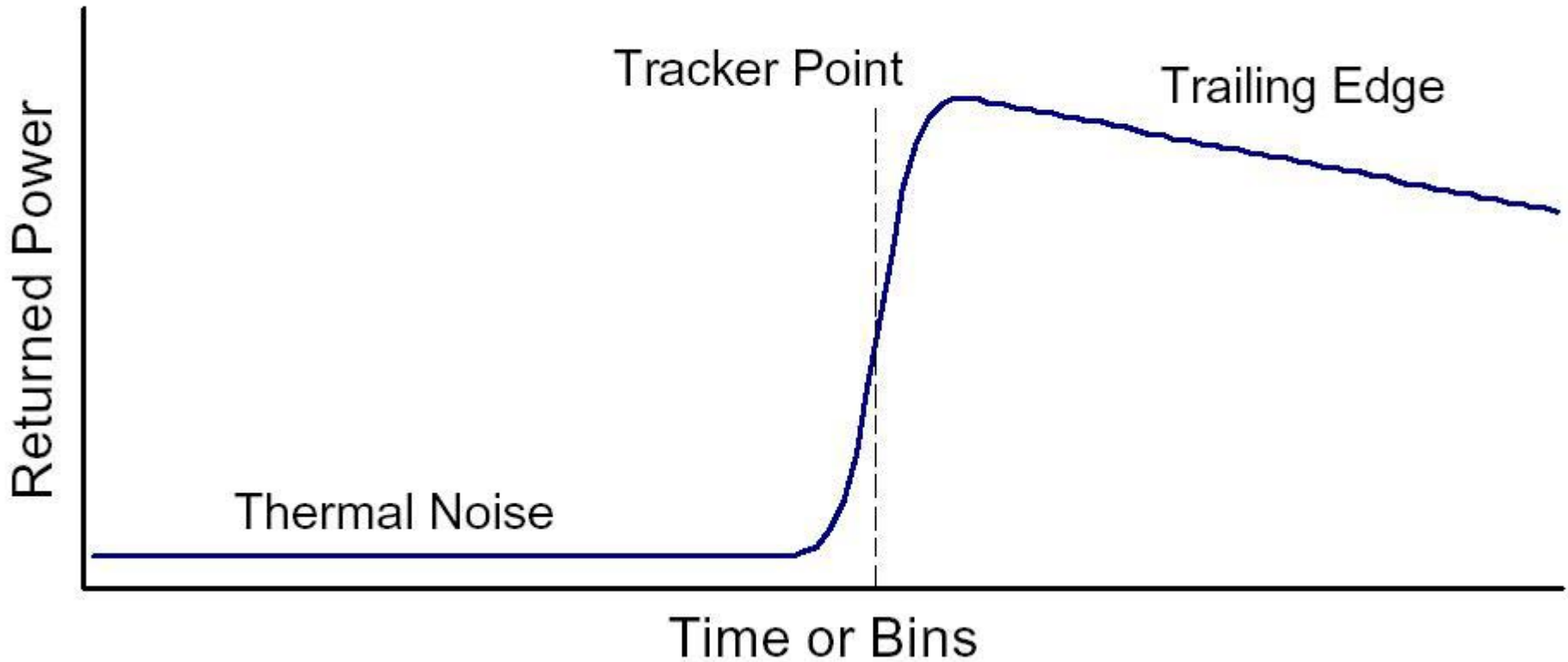


# Challenges

- Problematic areas
  - Shallow water
  - Islands and close to the shore
  - Areas with sea ice
- Retracking may solve some of these problems



# *Retracking*



# Orbits

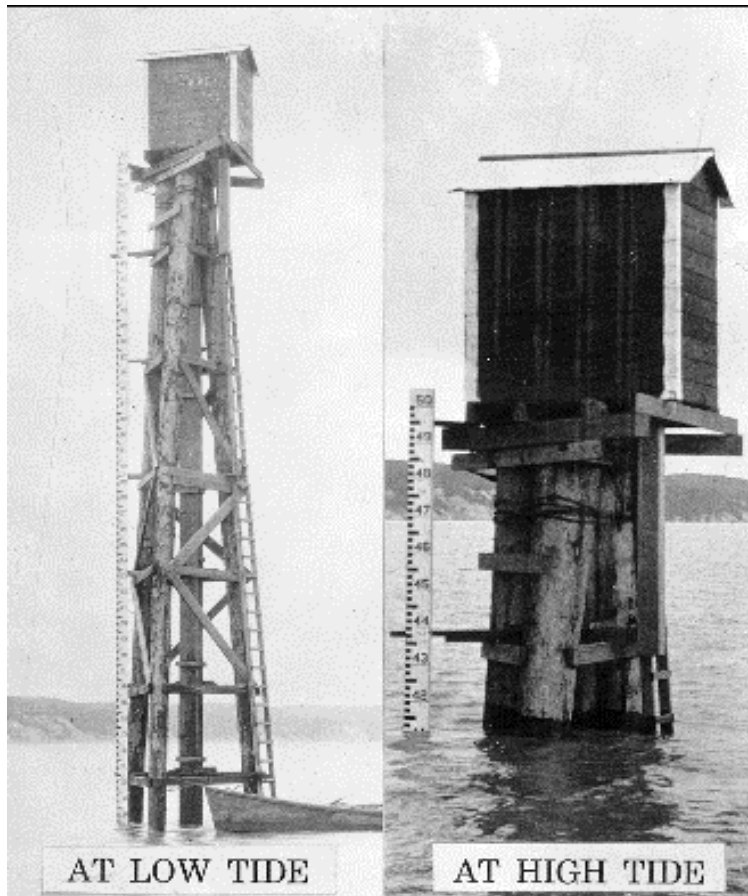
## Conflict of interests

Long repeat period, geodesy, good geographical/ spatial coverage

Short repeat period, oceanography, the same marine areas are surveyed repeatedly with short repeat periods, advantageous for study of time dependent features



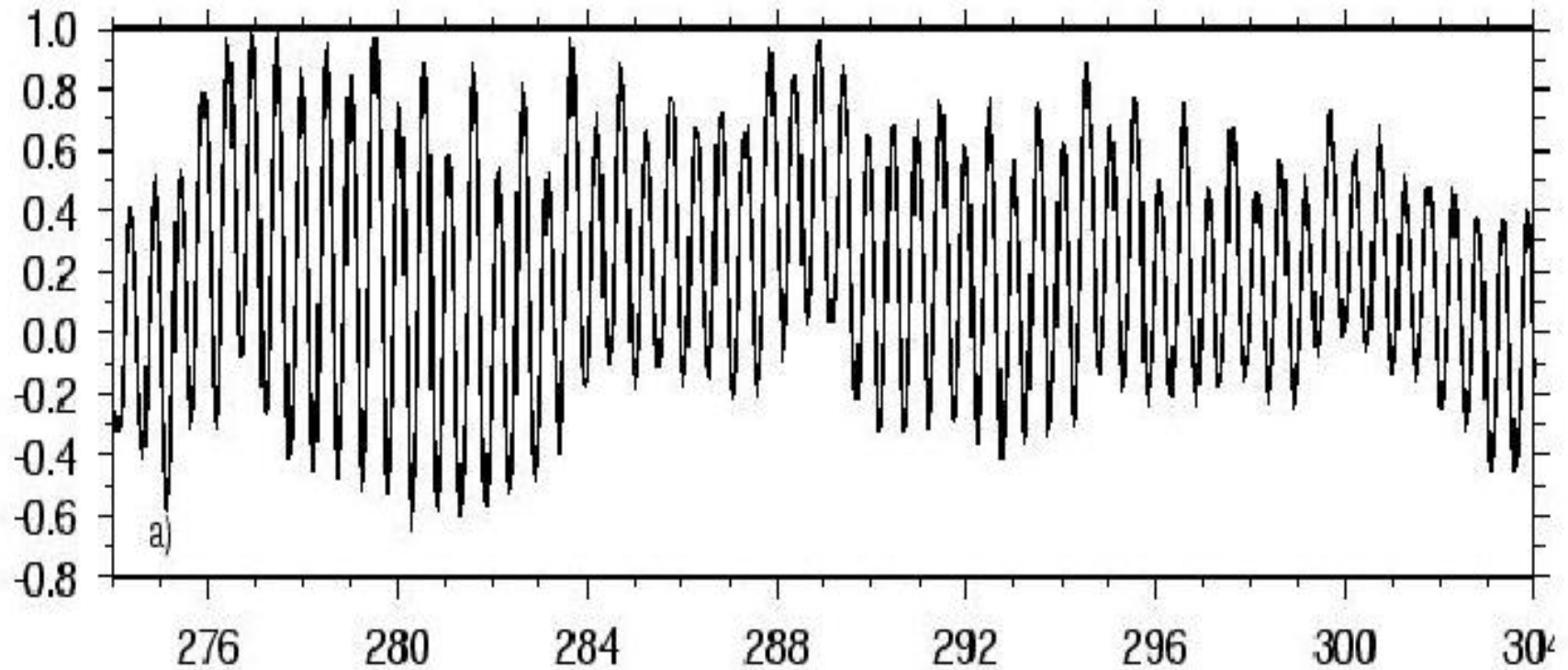
# Tide gauges



- Tide gauges measure the sea level relative to a nearby geodetic **benchmark**



# *Observed tide gauge*



# ***Harmonic constituents***

***Determined by the sun and the moon.***

***Most important constituents have been given names. Examples are:***

***M2: 2 cycles per lunar day***

***M1: 1 cycle per lunar day***

***S2: 2 cycles per solar day***

***Combination of frequencies: K1, O1, L2, N2 etc.***





# *Harmonic analysis*

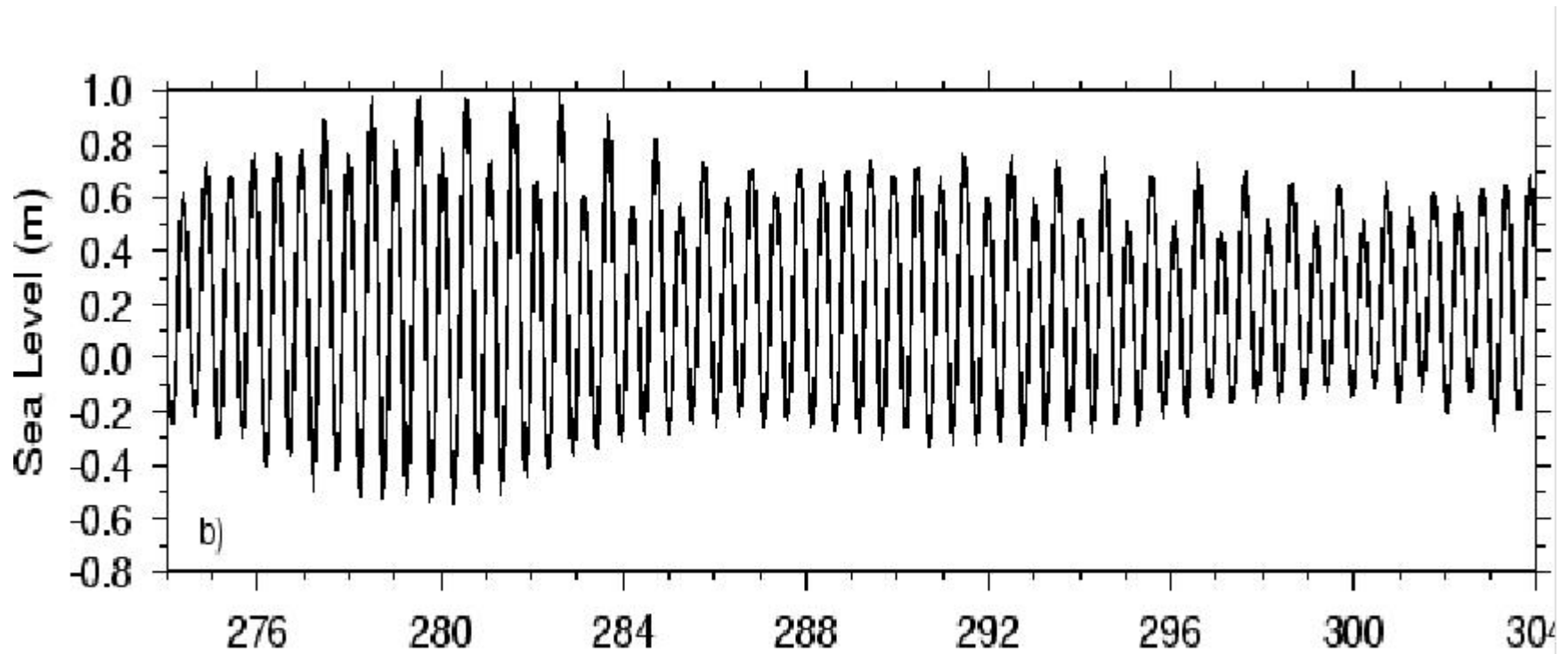
*Tide, approximated with a Fourier series*

*Known frequencies and time*

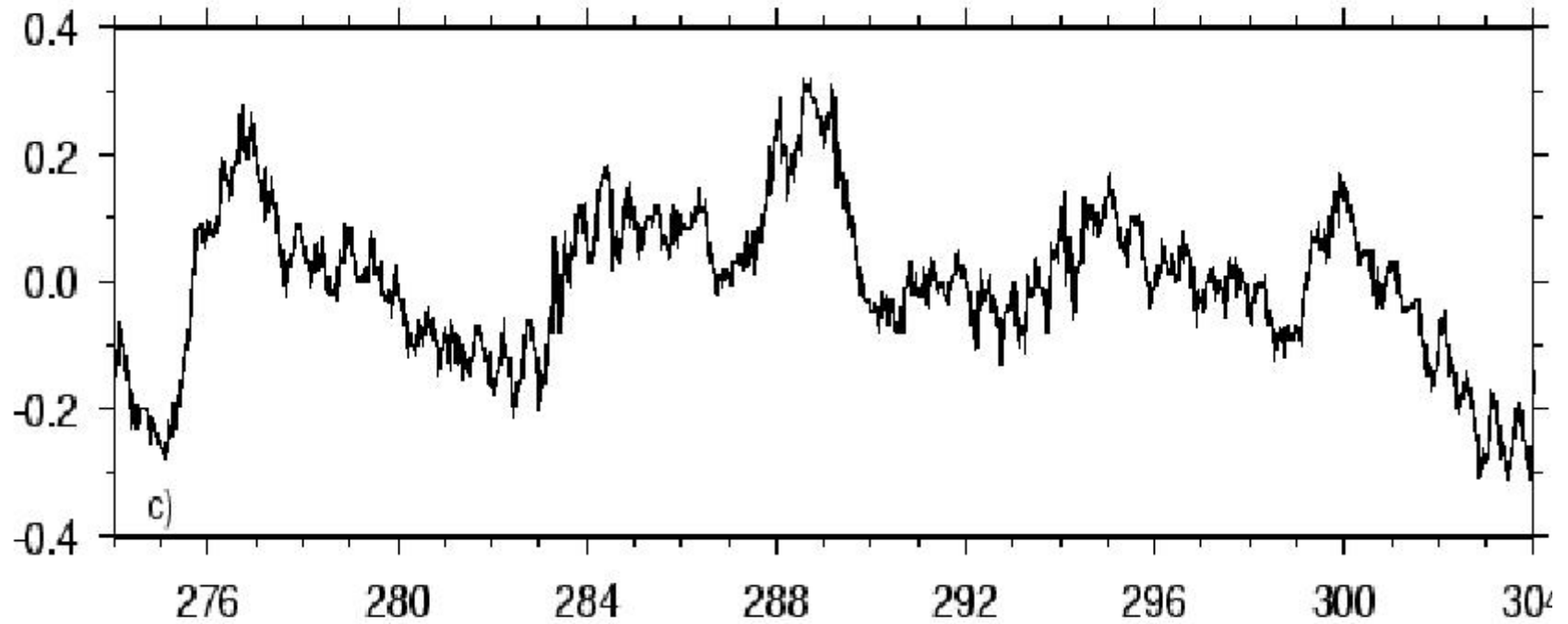
*Solve for amplitude and phase by least squares*



# *Predicted tide gauge*



# *Residual tide gauge*



# ***Lowest Astronomical Tide, LAT***

- Tide gauge observations***
- Harmonic analysis***
- Harmonic constituents***
- These harmonic constituents defines LAT,  
Lowest Astronomical Tide***
  - This is the lowest tide that can be expected to occur under  
average meteorological conditions***



# ***Variation with time***

***Changes relative to geodetic benchmark***

***Changes in sea level?***

***Land uplift?***

***Combination of both?***

***GPS needed to determine the cause of the change***



# ***Practical implementation of height reference systems***

## ***Land areas***

***Geoid/quasigeoid***

***Fit model to GPS/leveling***

***Example of the method used in Norway***

***Iterative procedure***

***Example of adjustments***

***Limitations and future work***



# *Geoid, quasigeoid, heightreference surface*

## □ Geoid:

- Equipotential surface,  $W=\text{const}$ . Requires knowledge about the earth's density. GPS + geoid => Orthometric heights.

## □ Quasigeoid:

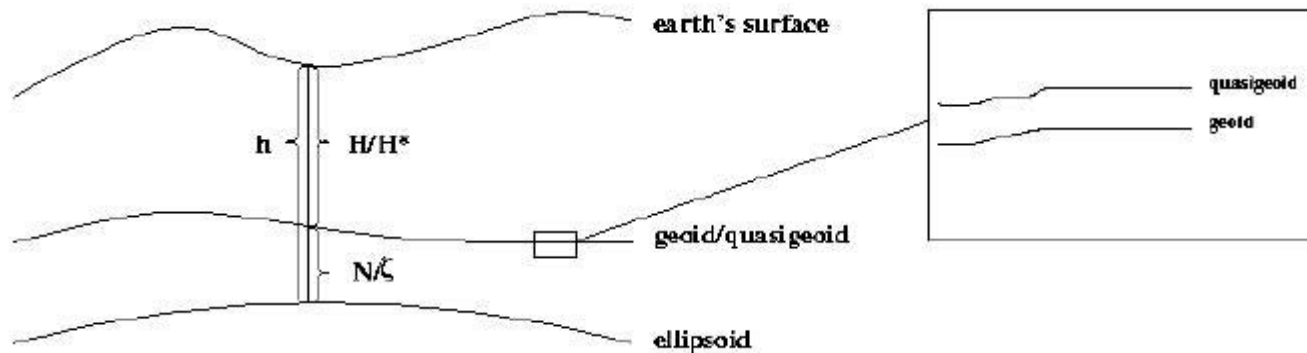
- Geoid-like surface but no equipotential surface. No knowledge about the earth's density required. GPS+quasigeoid => Normal heights.

## □ Height-reference surface:

- Surface adjusted to a vertical datum by GPS/leveling data. GPS + height-reference surface => Heights in desired datum.



# Geoid/quasigeoid



$$h = H + N = H^* + \zeta$$

**H:** Orthometric height

**N:** Geoidal undulation

**H\*:** Normal height

**ζ:** Height anomaly



# *Iterative procedure*

Given a model N

Given a set of GPS/leveling data

Adjusting this data set to model N gives us a new model N+1

The adjustment is typically performed by adjusting the existing model in a limited geographical area where there are discrepancies between the old model and the GPS/leveling dataset. As more GPS measurements in leveling points are made available new local adjustments can be performed.



# *List of models*

**HREF + GPS => Normal-heights**

**VREF + GPS => Orthometric heights**

**All models are derived from the NKG96 quasigeoid**

**NKG96**



**HREF1996**

**1998**

**1999a,b**

**2000a,b,c**

**2001a,b,c**

**2002a,b,c,d,e**

**2003a,b,c,d**



**NKG96n**



**VREF1996**

**1998**

**1999a,b**

**2000a,b,c**

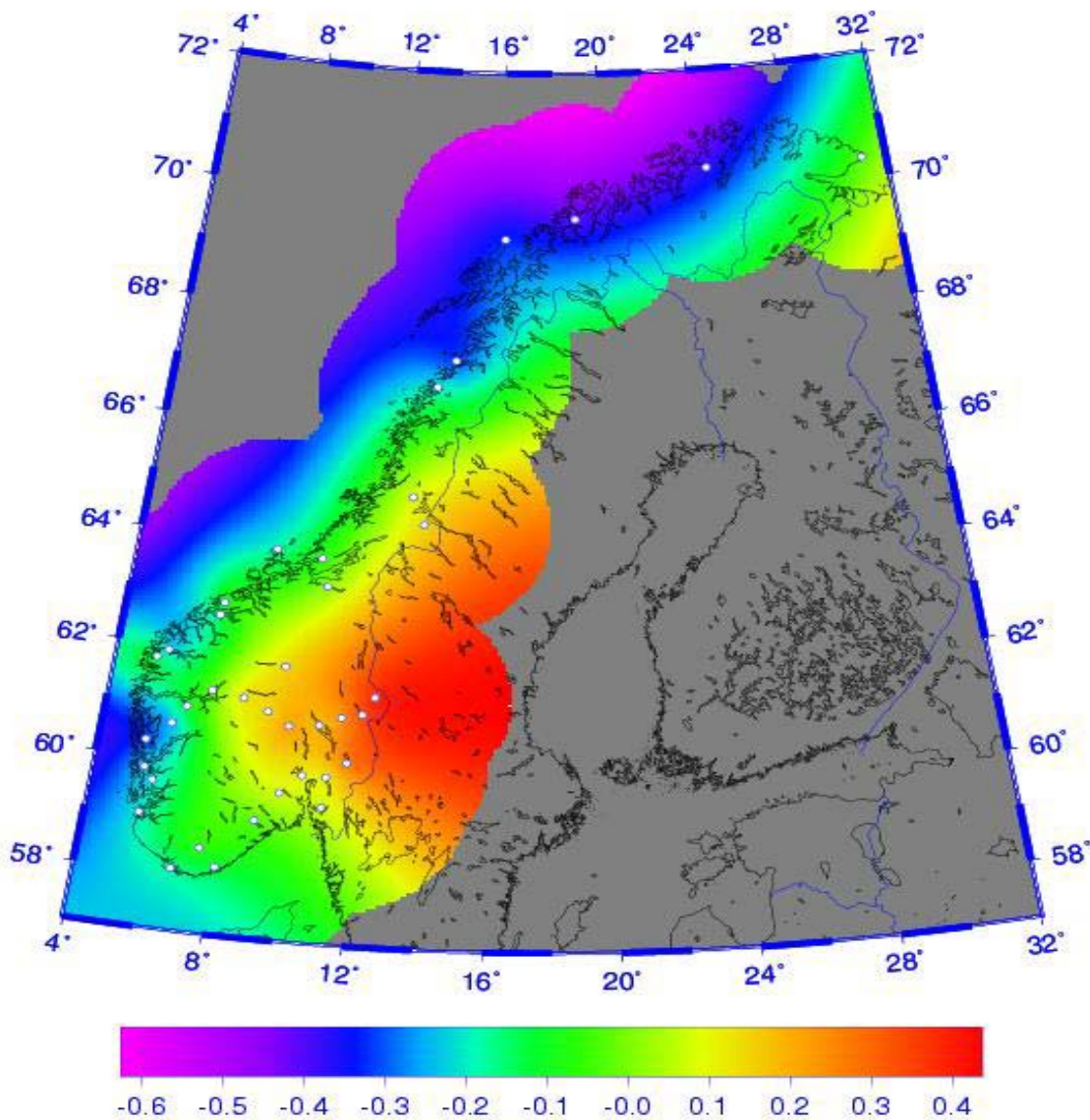
**2001a,b,c**

**2002a,b,c,d,e**

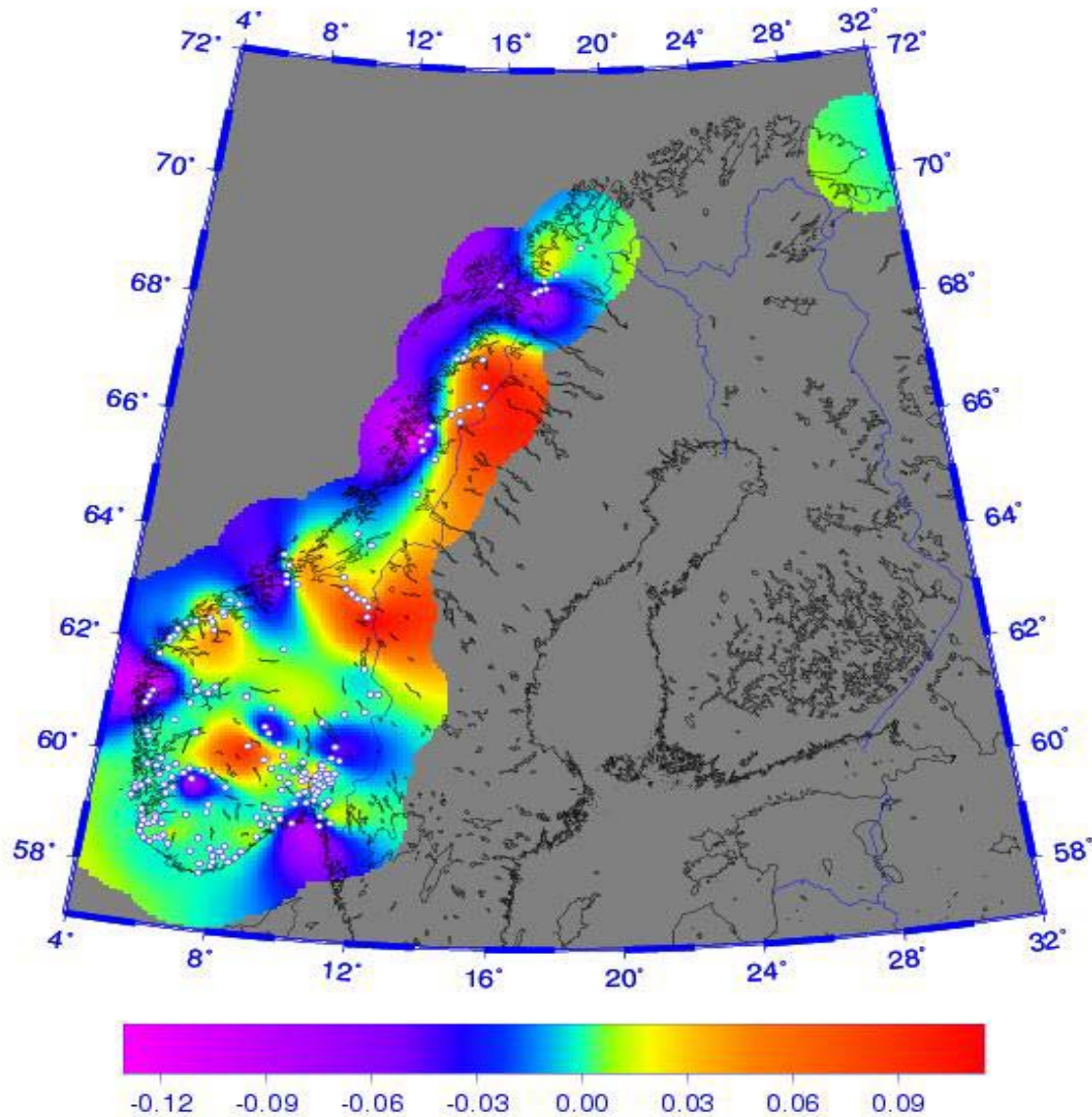
**2003a,b,c,d**



# HREF1996- NKG96 (m)



# HREF1998- HREF1996 (m)



# Quality control of the models

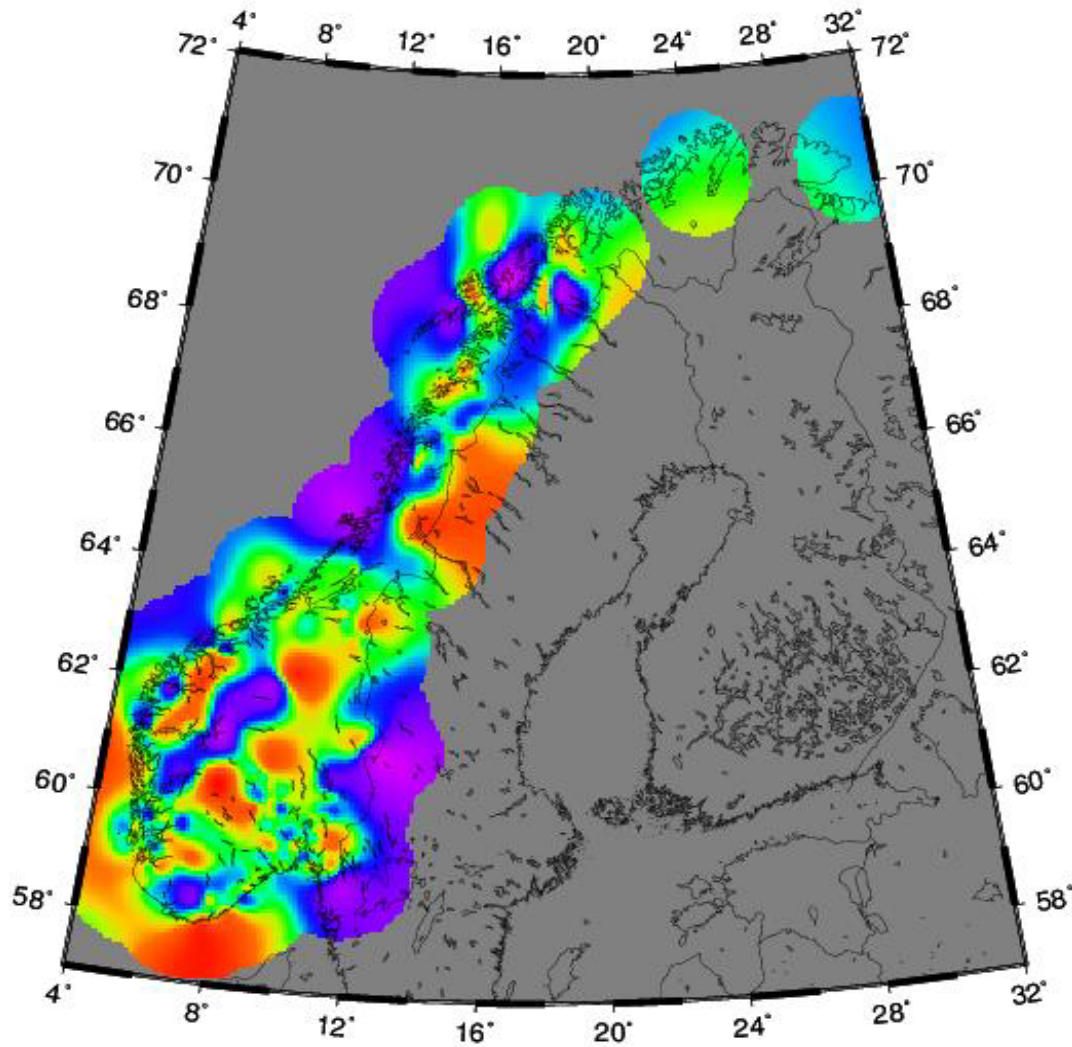
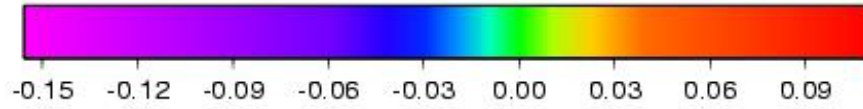
**Compare the models with GPS/leveling data that has not been used in the adjustment procedure.**

**The next table shows this comparison for a few Norwegian models (m).**

Model	N	mean	min	max	rms	stdv
NKG96	915	-.029	-.467	.378	.161	.158
HREF1996	877	.002	-.165	.150	.047	.047
HREF1998	787	-.001	-.164	.126	.043	.043
HREF2000b	718	-.004	-.156	.110	.028	.028



# $N_{GPS/LEV} - N_{HREF2000b}$ (m)



# Challenges

- Improve the uneven distribution of GPS measurements in leveling points*
- Improve the methods (theory, data coverage and data quality)*
- Investigate alternative methods*
- Land uplift*
- Time dependency*
- Point positioning (relative or absolute)*
- Impact of new height datum*
- Impact of improved geoid models*



# *Marine reference surfaces*

- Mean Sea Surface, MSS*
- Geoid*
- Surface given by LAT, the Lowest Astronomical Tide*





# *Mean Sea Surface*

Determined by satellite altimetry

Desired by the offshore industry

Not well determined in shallow areas, near land and islands or areas with sea ice

Mean for a given period which may or may not be representative for the "real" mss



# *Marin Geoid*

Needed by oceanography for an optimal use of satellite altimetry in determining the Mean Dynamic topography, ocean currents etc

The marine geoid may also be used as an interpolation surface between satellite altimetry ground tracks or to bridge the altimetric mss with mss from tide gauges



# ***GOCINA and OCTAS***

***Two closely related research projects, GOCINA, Geoid and Ocean Circulation In the North Atlantic funded by EU and OCTAS, Ocean Circulation and Transport Between North Atlantic and the Arctic Sea, funded by the Norwegian research Council.***

***Main goal: Determine the mean dynamic topography, MDT***

***See <http://www.gocina.dk> and  
<http://www.octas.statkart.no>***

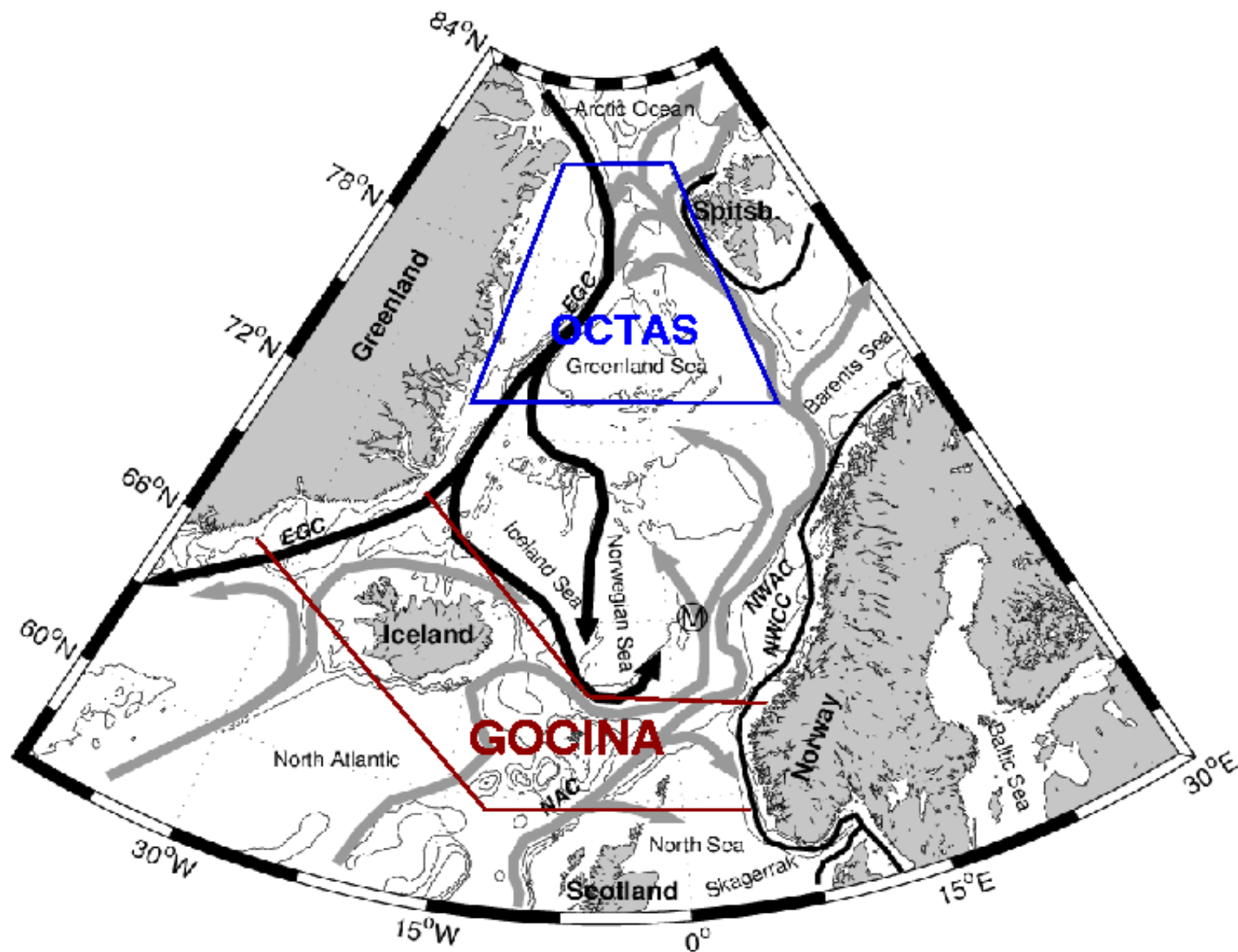


# *Determination of MDT*

To obtain this goal a dedicated airborne gravity campaign was performed in 2003, creating a gravity reference field for adjustment of marine gravity data. This improved gravity dataset will be used to compute a state of the art geoid for the study area. Combined with the best MSS from Satellite Altimetry the MDT can be determined. A combined simultaneous determination of all the 3 fields, Geoid, MSS and MDT, is also a part of the two projects.



# Study area



# ***Lowest Astronomical Tide, LAT***

***Tide gauges, harmonic analysis***

***Limited number of tide gauges along the coast  
where LAT is determined***

***MSS and tides from Satellite Altimetry determines  
a LAT surface for marine offshore areas***



# ***Suggestion on how to compute a LAT model***

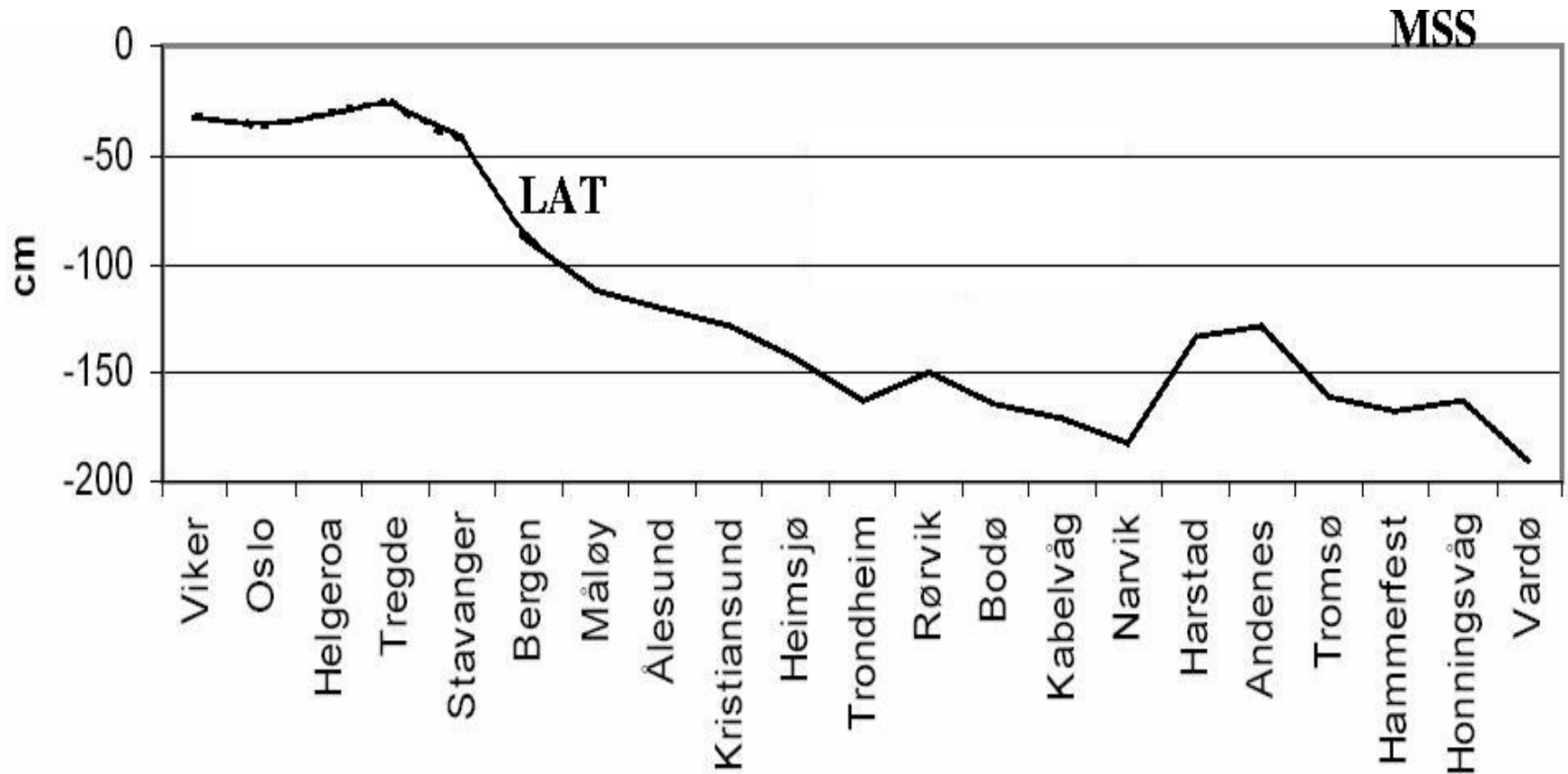
***Use a discrete number of tide gauges either alone or in combination with a LAT model from altimetry to compute a LAT model in a similar way as when adjusting geoid models to GPS/leveling This will establish a LAT model.***

***Having established this model then GPS in combination with a geoid model will determine LAT relatively to this GPS point.***

***It remains to be seen if this method is feasible or not!***



# LAT and MSS at Norwegian Tide Gauges





# *Other marine reference systems*

- HAT, Highest Astronomical Tide***
  - Navigation*
- 0-level for heights***
  - Norway, NN1954, do not coincide with MSS*
- 0-level for depths***
  - May deviate from LAT in areas with a very small tidal signal*
- Local systems***



# *Future improvements*

Satellite missions like CHAMP, GRACE, GOCE and their follow ons will improve our knowledge of the gravity field both spatially and time wise

Combined with local data (gravity etc.) new improved geoid models will be derived.

This may lead to a unified global height system.



# Summary

- Geoid***
- Leveling***
- Satellite altimetry***
- Marine and land reference surfaces***
  - Adjustment to national height datums***
  - Adjustment to tide gauge data***
- Future improvements***

